

AP Calculus BC

$$1) \int \frac{x+4}{x+6} dx = \int \left[1 - \frac{2}{x+6} \right] dx$$

$$= \left[x - 2 \ln|x+6| + C \right]$$

$$3) \int_{-1}^1 x^{-1/3} dx = \int_{-1}^0 x^{-1/3} dx + \int_0^1 x^{-1/3} dx$$

$$\lim_{b \rightarrow 0^-} \int_{-1}^b x^{-1/3} dx + \lim_{b \rightarrow 0^+} \int_b^1 x^{-1/3} dx$$

$$\lim_{b \rightarrow 0^-} \left[\frac{3}{2} x^{2/3} \right]_{-1}^b + \lim_{b \rightarrow 0^+} \left[\frac{3}{2} x^{2/3} \right]_b^1$$

$$\lim_{b \rightarrow 0^-} \left[\frac{3}{2} b^{2/3} - \frac{3}{2} \right] + \lim_{b \rightarrow 0^+} \left[\frac{3}{2} - \frac{3}{2} b^{2/3} \right]$$

$$0$$

$$5) \int_{-\infty}^{\infty} 16x^7 e^{-x^8} dx$$

$$\lim_{a \rightarrow -\infty} \int_a^0 16x^7 e^{-x^8} dx + \lim_{b \rightarrow \infty} \int_0^b 16x^7 e^{-x^8} dx$$

$$\lim_{a \rightarrow -\infty} \left[-2e^{-x^8} \right]_a^0 + \lim_{b \rightarrow \infty} \left[-2e^{-x^8} \right]_0^b$$

$$\lim_{a \rightarrow -\infty} \left[-2 + \frac{2}{e^{a^8}} \right] + \lim_{b \rightarrow \infty} \left[-\frac{2}{e^{b^8}} + 2 \right]$$

$$0$$

Long Division & Review

$$2) \int \frac{t^3-4}{t+2} dt$$

$$-2 \begin{array}{rrrr} 1 & 0 & 0 & -4 \\ & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & -12 \end{array}$$

$$\int \left[t^2 - 2t + 4 - \frac{12}{t+2} \right] dt$$

$$\left[\frac{1}{3} t^3 - t^2 + 4t - 12 \ln|t+2| + C \right]$$

$$4) \int_{14}^{\infty} \frac{4}{v^2-v} dv$$

$$\lim_{b \rightarrow \infty} \int_{14}^b \frac{4}{v(v-1)} dv$$

$$\frac{4}{v(v-1)} = \frac{A}{v} + \frac{B}{v-1}$$

$$A(v-1) + Bv = 4$$

$$\frac{v=1}{B=4}$$

$$\frac{v=0}{-A=4}$$

$$A=-4$$

$$\lim_{b \rightarrow \infty} \int_{14}^b \left[\frac{4}{v-1} - \frac{4}{v} \right] dv$$

$$\lim_{b \rightarrow \infty} \left[4 \ln|v-1| - 4 \ln|v| \right]_{14}^b$$

$$\lim_{b \rightarrow \infty} \left[4 \ln \frac{v-1}{v} \right]_{14}^b$$

$$\lim_{b \rightarrow \infty} \left[4 \ln \frac{b-1}{b} - 4 \ln \frac{13}{14} \right]$$

$$6) \int_5^{11} \frac{18x}{9x^2+6} dx$$

$$\int_5^{11} 18x(9x^2+6)^{-1} dx$$

$$\left[\ln(9x^2+6) + C \right]_5^{11}$$

$$\ln(1095) - \ln(231)$$

$$8) \int_1^{e^4} x^3 \ln x dx$$

$$u = \ln x \quad v = \frac{1}{4}x^4$$

$$du = \frac{1}{x} dx \quad dv = x^3 dx$$

$$\frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^3 dx$$

$$\left[\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 \right]_1^{e^4}$$

$$\left[\frac{1}{4}e^{16}(4) - \frac{1}{16}e^{16} \right] - \left[0 - \frac{1}{16} \right]$$

$$7) \int x \sin\left(\frac{1}{5}x\right) dx$$

$$\frac{D}{x}$$

$$\frac{I}{\sin\left(\frac{1}{5}x\right)}$$

$$1$$

$$-5 \cos\left(\frac{1}{5}x\right)$$

$$0$$

$$-25 \sin\left(\frac{1}{5}x\right)$$

$$-5x \cos\left(\frac{1}{5}x\right) + 25 \sin\left(\frac{1}{5}x\right) + C$$

$$9) \int 9x^3 e^{3x} dx$$

$$\frac{D}{9x^3}$$

$$\frac{I}{e^{3x}}$$

$$-27x^2$$

$$\frac{1}{3}e^{3x}$$

$$+54x$$

$$\frac{1}{9}e^{3x}$$

$$-54$$

$$\frac{1}{27}e^{3x}$$

$$0$$

$$\frac{1}{81}e^{3x}$$

$$3x^3 e^{3x} - 3x^2 e^{3x} + 2x e^{3x} - \frac{54}{81} e^{3x} + C$$

$$10) \int \frac{10r^4}{\sqrt{1-r^5}} dr$$

$$\int 10r^4 (1-r^5)^{-1/2} dr$$

$$u = 1-r^5$$

$$du = -5r^4 dr$$

$$\frac{du}{-5r^4} = dr$$

$$\int -2u^{-1/2} du$$

$$-4u^{1/2} + C$$

$$\boxed{-4(1-r^5)^{1/2} + C}$$

$$11) \int_0^{\pi/6} \cos^{-8} 2x \sin 2x dx$$

$$u = \cos 2x$$

$$u(0) = 1$$

$$du = -2 \sin 2x dx$$

$$u\left(\frac{\pi}{6}\right) = \cos \frac{\pi}{3} = \frac{1}{2}$$

$$-\frac{1}{2 \sin 2x} = dx$$

$$-\frac{1}{2} \int_1^{1/2} u^{-8} du$$

$$-\frac{1}{2} \left[-\frac{1}{7} u^{-7} \right]_1^{1/2}$$

$$-\frac{1}{2} \left[-\frac{1}{7} \left(\frac{1}{2}\right)^{-7} + \frac{1}{7} \right]$$